Influence of Chemical Kinetic and Turbulent Transport Coefficients on Afterburning Rocket Plumes

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The influence of chemical kinetic and turbulent transport coefficients on electrical properties of solid-propellant plumes is demonstrated via predictions of transverse radar attenuation at sea level and for an altitude of 28 kft. A model of parallel turbulent mixing and nonequilibrium combustion and ionization is used to determine whether varying these coefficients over their ranges of uncertainty significantly affects predicted attenuations. An explicit finite difference technique is utilized to solve the free shear layer equations, and the Donaldson/Gray (D/G) modification of the Prandtl eddy viscosity formulation is used to describe the turbulent transport of momentum, mass, and energy. The results show that 1) large differences in predicted attenuation levels can result from the abovementioned variations, 2) the D/G model gives better agreement with sea level attenuation data than the Ting/Libby modification of the Prandtl model, and 3) a better laboratory determination of the rate coefficient for the dominant charge-producing reaction, $K + Cl \rightarrow K^+ + Cl^-$, is of particular importance if more accurate attenuation predictions are to be made. It is concluded that realistic calculations of solid-propellant exhaust plume electrical properties must take chemical kinetics into account. The assumption that local thermochemical equilibrium is established can lead to results which are considerably in error.

Nomenclature

a = radar attenuation
b = width of mixing region
k = rate coefficient
K = Ting/Libby eddy viscosity coefficient, developed region
K_i = Ting/Libby eddy viscosity coefficient, initial region
$\bar{K}$ = Donaldson/Gray eddy viscosity coefficient
K_p = equilibrium constant
K_p' = defined by Eq. (4)
Le_T = turbulent Lewis number
$M_{1/2}$ = Mach number at $r = r_{1/2}$
M = Mach number
n_e = electron density
Pr_T = turbulent Prandtl number
r = radial distance from plume axis
$\bar{r}_{1/2}$ = value of η [Eq. (2c)] corresponding to $r = r_{1/2}$
r_i = radial distance to boundary of inner mixing zone in core region
u = axial velocity
$\dot{w}$ = species production rate
ϵ = eddy diffusivity for momentum transfer
ν_e = electron-neutral collision frequency
μ = eddy viscosity for momentum transfer
ρ = density
ω = signal frequency, rad/sec

Subscripts

D/G = Donaldson/Gray
e,j,o = edge of mixing region, nozzle exit plane, axis
T/L = Ting/Libby
∞ = freestream
1/2 = at r where $u = (u_0 + u_e)/2$

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Introduction

PREDICTION of exhaust plume electrical properties is a necessary step in the design of missiles guided via radar signals passing through the plume.[1] These properties are important to the determination of all radar interference effects (attenuation, reflectivity, modulation)—which must be known to make tradeoff studies with, e.g., propulsion and guidance system requirements. The objective of this study was to determine whether varying reaction rates and turbulent transport coefficients—input data to a model of parallel mixing and nonequilibrium afterburning—significantly affects the levels of predicted radar attenuation (integrated electron density) transverse to solid-propellant plumes. Because rocket motors operate fuel-rich these plumes are characterized by large initial mole fractions of CO and H_2, which react with ambient air to increase temperatures and ionize the alkali metals present as impurities in all solid propellants.‡ Therefore, the prediction of exhaust plume electrical properties depends on a knowledge of combustion and ionization rates, as well as the rate of turbulent transport. The present state-of-the-art in predicting these properties is such that "good" agreement is achieved when transverse radar attenuation data and calculations are within a factor of two (in db) under sea level static conditions. This implies that predicted and actual electron densities are within a factor of two. Only recently has simulated altitude wind-tunnel data become available for comparison with theory, and no good comparison between in-flight data and calculations of plume electrical properties has yet been made.

Previous studies[2,3] have already demonstrated the importance of nonequilibrium chemistry (and consequently the need for good rate coefficient data) in determining plume properties at moderate altitudes (say from 20–50 kft). Since

‡ Chemi-ionization of unburned hydrocarbons, another mechanism of electron and ion production in afterburning plumes, has been discussed previously[2] in relation to liquid propellant exhausts. This mechanism is generally not important for typical solid-propellant systems.

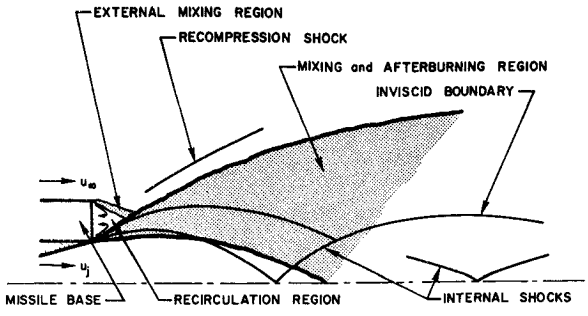

Fig. 1 Schematic of low-altitude exhaust plume flowfield.

many of the rate coefficients (k's) have wide ranges of uncertainty, calculated electrical properties can be considerably in error. Thus, it is very important to know 1) which reactions dominate electron production and depletion rates and 2) how electron densities are affected when k's for these reactions are varied over their ranges of uncertainty. Similar problems arise in describing turbulent transport coefficients (for momentum, mass, and energy) for flows with combustion. Although many phenomenological models of turbulent transport coefficients have been suggested, most have been developed for nonreacting environments.[4-6] Only recently has an effort been made to determine these coefficients for flows with combustion,[7-9] and it is not yet clear how the results of these studies can best be applied to the rocket exhaust plume problem. It is therefore evident that the turbulent transport coefficients used in plume calculations also require a parametric investigation.

A low-altitude exhaust plume flowfield is depicted in Fig. 1. In principle, calculations of electron densities require an analysis of the strong viscous-inviscid interaction problem for turbulent, chemically reacting free shear layers, including the important effects of recirculating flow in the missile base region. A complete gas dynamic model of this complex flowfield suitable for use in interpreting radar attenuation data has yet to be developed,§ and, for the present, a parallel mixing model must suffice. In qualitative terms, it is reasonable to assume that such a mixing model will be correct for static ($u_\infty = 0$) external conditions provided the nozzle is not highly underexpanded, i.e., for ratios of nozzle exit pressure to ambient pressure less than about 5.¶ In addition, since electron production occurs primarily in the mixing and afterburning regions for low altitude plumes,[11] the influence of the shock structure on electron density should be unimportant except in a small region near the plume axis (i.e. the strongest part of the shock).

In the present study attenuation predictions were made for $u_\infty = 0$, at sea level and 28 kft, since they can be readily compared with simulated altitude (wind-tunnel) data taken in a static environment as such data become available. The initial calculations used 1) recommended[12] k's determined either from laboratory data or, where such data were not available, estimates based on k's for similar reactions and 2) an eddy viscosity model consistent with some nonreactive turbulent mixing data.[13] The most important rate coefficients and turbulent mixing rates were then varied over their estimated ranges of uncertainty to determine their influence on predicted attenuation. Since only a limited number of computer experiments was possible, we concentrated on varying the k's for those reactions already identified[14,15] to play the most important roles in determining local electron densities.

§ Significant progress toward the development of such a model has been made by Edelman and Weilerstein.[10]

¶ For much larger pressure ratios the curvature of the inviscid plume boundary must be taken into account.

Nonequilibrium Afterburning Model

The key assumptions employed in the present model (described in more detail elsewhere[2,11]) are: 1) pressure is constant throughout the flow, 2) there are no shocks in the flow, and 3) the aluminum oxide particles behave as a gas, i.e., they contribute to the density and molecular weight of the mixture in the same manner as do all other species. Particle temperature and velocity lags are assumed to be zero and heterogeneous chemistry is assumed negligible. The governing set of partial differential equations, i.e., the usual turbulent free shear layer equations for a chemically reacting system[16] were solved via an explicit finite-difference technique. The computer program[17] used to obtain the present results was run on a Univac 1108.

The chemical reaction mechanism and recommended k's (with their estimated error bounds) used in this study are given in Table 1.** Obviously, many more reactions can be written incorporating the 17 important species present at the nozzle exit plane (cf. Table 2). However, by including only reactions which make a significant contribution to the plume electron densities,[14,15] we are left with the 19 reactions shown in Table 1. It must be emphasized that reaction mechanisms are unique to specific propellant systems—there is no "universal" mechanism. However, for solid-propellant exhausts with nozzle exit plane compositions similar to those given in Table 2, the mechanism given here should suffice.

We must emphasize that actual k's cannot be inferred from comparisons between the predictions made in this study and transverse attenuation data. Clearly, the uncertainties in all the k's and turbulent transport coefficients are such as to preclude obtaining a unique answer (i.e., a single set of coefficients which gives agreement between theory and experiment). Therefore, the k's given in Table 1 will continue to be recommended values based on the available laboratory data.

Because insufficient data on flows with combustion are available there is considerable uncertainty as to the most realistic eddy viscosity model to use in afterburning plume calculations. The "standard" model used in this study is the Donaldson/Gray[13] (D/G) modification of the Prandtl model,[18] which has been used to account for some high-temperature flow data.[19] The D/G eddy viscosity is expressed as:

$$\mu = \rho\epsilon = \bar{K}(r_{1/2} - r_i)\rho|u_o - u_e|/2 \qquad (1)$$

where $r_i > 0$ in the initial (core) region, before the mixing zone intersects the axis, and $r_i = 0$ in the developed region. The coefficient $\bar{K}$ is a function of the Mach number at $r = r_{1/2}$. Smoot[20] has also recommended use of the D/G model in calculations for particle-laden ducted flows with application to air-augmented solid-propellant systems. Peters[21] indicates that, in the initial region, the value of $\bar{K}$ should be lower than that recommended by Donaldson and Gray[13] by a factor of about 1.5 based on his ". . . experience in correlating a similar integral method with low speed experiments . . ." Note that the D/G model assumes no radial variation in eddy diffusivity (ϵ). However, recent measurements by Peters et al.[22] demonstrate that, for nonreactive air-air and hydrogen-air systems, the eddy diffusivity does vary in a radial direction; this variation could not be simply related to the radial density profile (cf. Ting/Libby[23] model described below). No corresponding measurements are available for flows with combustion.

For most of the calculations reported herein, the D/G values of $\bar{K}$ as a function of Mach number at the half radius were used. Several calculations were also made in which $\bar{K}$ was reduced by a factor of 1.5 in both the initial and developed regions. In addition, the results of a single calculation

** All k's were taken from Ref. 12 which fully discusses how the available laboratory data were interpreted to arrive at recommended values. In the Appendix we discuss the sources of rate coefficient data for the charged species reactions.

Table 1 Chemical reaction mechanism for afterburning solid-propellant plumes[a]

	H_2/CO combustion	Recommended forward rate coefficients[b] (cm-molecule-sec units)	Error bounds[c] Upper	Lower
1.	$O + O + M \rightleftarrows O_2 + M$	$1 \times 10^{-29}\, T^{-1}$	10	10
2.	$O + H + M \rightleftarrows OH + M$	$1 \times 10^{-29}\, T^{-1}$	10	10
3.	$H + H + M \rightleftarrows H_2 + M$	$1 \times 10^{-29}\, T^{-1}$	5	5
4.	$H + OH + M \rightleftarrows H_2O + M$	$1 \times 10^{-28}\, T^{-1}$	10	10
5.	$CO + O + M \rightleftarrows CO_2 + M$	$5 \times 10^{-29}\, T^{-1} \exp -4000/RT$	3	30
6.	$OH + OH \rightleftarrows H_2O + O$	$1 \times 10^{-11} \exp -1000/RT$	5	5
7.	$OH + H_2 \rightleftarrows H_2O + H$	$4 \times 10^{-11} \exp -5500/RT$	3	3
8.	$O + H_2 \rightleftarrows OH + H$	$3 \times 10^{-11} \exp -8200/RT$	5	5
9.	$H + O_2 \rightleftarrows OH + O$	$3 \times 10^{-10} \exp -16500/RT$	1.5	1.5
10.	$CO + OH \rightleftarrows CO_2 + H$	$5 \times 10^{-13} \exp -600/RT$	5	5
	Reactions of Cl-containing species			
11.	$H + Cl + M \rightleftarrows HCl + M$	$3 \times 10^{-29}\, T^{-1}$	10	30
12.	$Cl + H_2 \rightleftarrows HCl + H$	$1.5 \times 10^{-10} \exp -5000/RT$	5	5
13.	$H_2O + Cl \rightleftarrows HCl + OH$	$5 \times 10^{-11} \exp -19000/RT$	30	30
14.	$OH + Cl \rightleftarrows HCl + O$	$3 \times 10^{-11} \exp -5000/RT$	30	30
	Neutral potassium			
15.	$^d K + HCl \rightleftarrows KCl + H$	3×10^{-10}	10	30
	Charge removal/production			
16.	$^e K^+ + e^- + M \rightleftarrows K + M$	$2 \times 10^{-22}\, T^{-1.5}$	3	3
17.	$^e K^+ + Cl^- \rightleftarrows K + Cl$	$1 \times 10^{-8}\, T^{-0.5}$	10	100
	Charge shifting			
18.	$^e Cl + e^- + M \rightleftarrows Cl^- + M$	3×10^{-30}	30	30
19.	$^e HCl + e^- \rightleftarrows H + Cl^-$	$1 \times 10^{-10} \exp -20000/RT$	300	10

[a] Refers in particular to solid-propellant systems having nozzle exit plane compositions similar to those in Table 2; assumes potassium is dominant alkali metal impurity.
[b] Rate coefficients taken from Ref. 12, which fully discusses the sources of the data.
[c] Taken from Ref. 12. The upper error bound is defined as the ratio of the probable upper limit of the rate coefficient to the estimated rate coefficient k itself at the temperature at which the rate coefficient is least accurately known. The lower error bound is the ratio of k to the estimated probable lower limiting value of the rate coefficient at this temperature.
[d] This reaction is very "fast" in a typical afterburning environment. It is therefore suggested that it be kept in equilibrium when making calculations. A special version of the program described in Ref. 17 keeps reaction (15) in equilibrium while allowing all other reactions to proceed via finite rates.
[e] Sources of rate coefficient data given in Appendix.

are shown in which attenuations calculated using both the D/G and Ting/Libby[23] (T/L) modifications of the Prandtl eddy viscosity model are compared with measurements.†† The T/L model is expressed in the form:

Initial Region

$$\mu = \rho\epsilon = K_i b|u_j - u_e|\rho(\rho_j/\rho)^2 \qquad (2a)$$

Developed Region

$$\mu = \rho\epsilon = K\bar{r}_{1/2}|u_o - u_e|\rho(\rho_o/\rho)^2(\eta/r)^2 \qquad (2b)$$

where

$$\eta^2 = 2\int_0^r (\rho_o/\rho)r'dr' \qquad (2c)$$

The transport of mass, momentum and heat were assumed to be similar, i.e., $Pr_T = Le_T = 1$, for most of these calculations. However since values of Pr_T ranging from about 0.6 to 0.85 have been measured[22,24,25] in nonreacting systems, some calculations were made for $Pr_T = 0.7$ and (to test for effect) 1.4. A turbulent Lewis number of unity was always used. This value has been measured for some nonreacting flows,[22] and appears to be reasonable for flows with combustion because the rate of heat release is closely tied to the transport of neutral species.

Calculation of Local Plume Properties

The initial (nozzle exit) conditions for the nonequilibrium afterburning calculations are given in Table 2. It should be noted that: 1) the mole fraction of fuel (H_2 and CO) for each system is large, e.g., system C contains over 50 mole percent fuel, indicating a large potential for energy release and the production of free radicals, 2) all systems contain aluminum oxide, and 3) most of the alkali metal impurity (potassium) is tied up as KCl.

Figure 2 shows effects of altitude, Pr_T and ϵ on temperature distributions along the plume centerline. Although afterburning causes a marked increase in temperature at sea level,

Table 2 Initial conditions for nonequilibrium afterburning calculations

	System A	System B	System C
Radius, ft	0.1	0.25 (sea level) 0.12 (28 kft)	0.14
Temperature, °K	2000	2000	1780
Composition, mole fraction			
H_2O	2.5(−1)[a]	2.8(−1)	1.4(−1)
N_2	8.8(−2)	1.3(−1)	8.0(−2)
CO	1.9(−1)	2.4(−1)	2.4(−1)
H_2	1.9(−1)	8.1(−2)	2.9(−1)
CO_2	4.2(−2)	1.8(−1)	2.0(−1)
HCl	1.8(−1)	5.0(−2)	1.5(−1)
Al_2O_3(s)	6.0(−2)	3.4(−2)	7.0(−2)
H	1.4(−3)	5.4(−3)	1.3(−3)
OH	4.0(−4)	1.2(−3)	5.5(−5)
O	2.5(−6)	6.1(−5)	1.0(−7)
Cl	9.0(−4)	2.7(−3)	3.6(−4)
O_2	1.0(−6)	1.2(−4)	7.5(−8)
KCl	5.0(−5)	6.0(−6)	3.2(−5)
K	8.0(−8)	3.3(−8)	9.6(−8)
K^+	3.0(−7)	5.3(−7)	9.7(−7)
Cl^-	3.0(−7)	5.2(−7)	1.0(−6)
e^-	7.0(−10)	1.3(−8)	2.0(−9)

[a] A(−B) ≡ A × 10^{-B}.

†† Other eddy viscosity models[5] could have been used for this comparison.

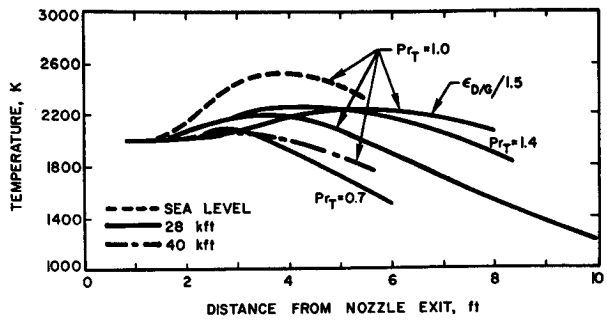

Fig. 2 Influence of turbulent mixing rate on centerline temperature distributions, System A; $\epsilon_{D/G}$ used except as noted.

the effect is considerably less as altitude is increased (cf. the effect of altitude on attenuation[3]). The influence of chemical kinetics on temperature is apparent since, if the flow were in equilibrium, the peak plume temperature would not be such a strong function of altitude. Note also that decreasing $\epsilon_{D/G}$ by a factor of 1.5 and varying Pr_T has an important effect on temperature. The results for $Pr_T = 1.4$ demonstrate what might happen if heat (and mass) transport were less than momentum transport in flows with combustion.

Typical electron density and temperature contours are shown in Fig. 3. Also shown is transverse radar attenuation,‡‡ computed from

$$a = 14 \int_r \frac{n_e/\nu_e}{1 + (\omega/\nu_e)^2}\, dr \text{ db} \qquad (3)$$

(n_e in cm^{-3}, ν_e in sec^{-1} and r in ft) and the electrical plume radius, the radius where 95% of the total transverse attenuation, computed from Eq. (3), is reached. The length of the electrical plume (i.e., axial distance to the location where the attenuation is negligibly small) is seen to be about 7 ft, a location where the temperature on the axis is still large, about 1700°K. In addition, it is noteworthy that the electrical plume radius does not extend to regions where temperatures are less than about 1200°K within 7 ft from the nozzle exit. This type of result indicates the temperature range for which accurate rate coefficient data are necessary, i.e., for this case, 1200°–2200°K, and also suggests that thermal plumes are likely to be larger than electrical plumes.

To show how some of the important charged species reactions depart from local equilibrium, we have computed K_p'/K_p, where for the reaction $A + B \rightleftarrows C + D$

$$K_p' = [C][D]/[A][B] \qquad (4)$$

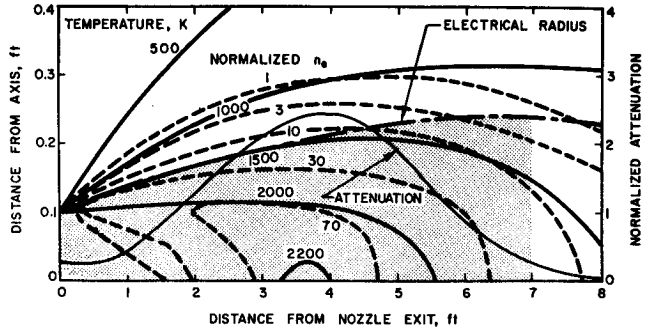

Fig. 3 Comparison between electrical and thermal plumes, System A, 28 kft. All rate coefficients as listed in Table 1 except $k_{17}/10$; $\epsilon_{D/G}$; $Pr_T = 1.0$.

‡‡ Electron-neutral collision cross sections were computed from the collision cross section data presented by Molmud.[26]

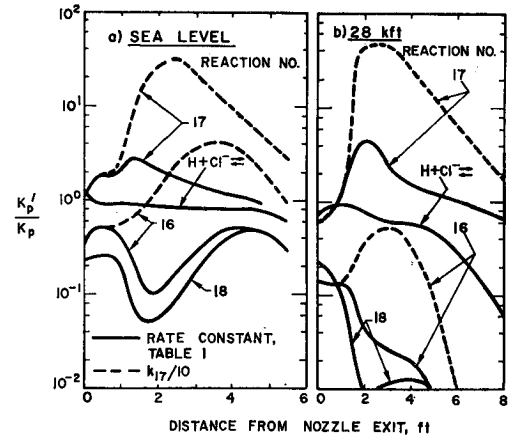

Fig. 4 Deviations of ionization reactions from local equilibrium, System A. All rate coefficients as in Table 1, except $k_{17}/10$ for all ---- curves; H + Cl^- ⇄ HCl + e^- is reverse of reaction (19).

and K_p is the equilibrium constant evaluated at the local temperature. When $K_p'/K_p = 1$, the reaction is in local equilibrium. Since the rate of species production is

$$\dot{w} \propto (1 - K_p'/K_p) \qquad (5)$$

the reaction is proceeding in a backward direction (to the left) when $K_p'/K_p > 1$. In Fig. 4a it can be seen that 1) even at sea level the reaction $K^+ + Cl^- \rightleftarrows K + Cl$ (reaction (17)] departs from equilibrium, 2) when k_{17}§§ is divided by 10 (within the error bounds noted in Table 1) departure from equilibrium is very drastic, 3) the reaction H + Cl^- ⇄ HCl + e^- is nearly in equilibrium, and 4) when the recommended value of k_{17} is used the forward step for $K^+ + e^- + M \rightleftarrows K + M$ is dominant throughout the plume, whereas if k_{17} is decreased by a factor of 10, there are some regions in the plume for which each reaction direction is dominant. In Fig. 4b it is observed that, as expected, these charged species reaction depart even further from equilibrium at 28 kft than they do at sea level.

By calculating rates of species production from individual reactions (taken from the computer output) it has been shown[14,15] that for typical solid-propellant exhausts at altitudes from sea level to 70 kft, the dominant mechanism of electron production is K + Cl → K^+ + Cl^- followed by H +

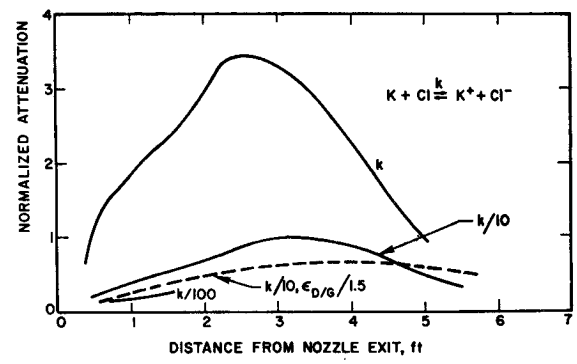

Fig. 5 Influence of ionization rate coefficient on attenuation calculations, sea level, System A. All rate coefficients as listed in Table 1, except as noted; $\epsilon_{D/G}$ except as noted; $Pr_T = 1.0$.

§§ Rate coefficient numbers refer to reaction numbers in Table 1.

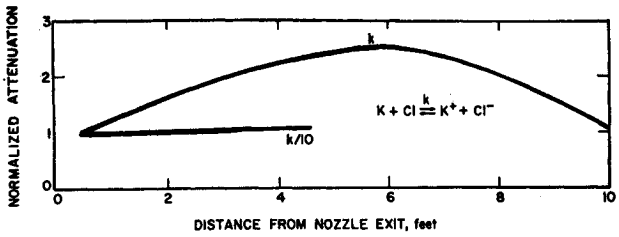

Fig. 6 Influence of ionization rate coefficient on attenuation calculations, sea level, System B. All rate coefficients as listed in Table 1, except as noted; $\epsilon_{D/G}$; Pr_T = 1.0.

$Cl^- \rightarrow HCl + e^-$, whereas the dominant electron depletion reactions are $K^+ + e^- + M \rightarrow K + M$ and electron attachment. Detailed reaction paths for H_2/CO combustion reactions have also been discussed.[14,15]

Radar Attenuation Calculations

Results of parametric attenuation calculations for systems *A*, *B*, and *C* are given in Figs. 5, 6, and 7. All calculations were made for a signal frequency of 10,000 MHz. It is reasonable to assume that very little of the uncertainty in these calculations is due to inaccuracies in the collision frequency calculations. Collision frequencies are accurately known because the electron-water molecule collision cross section,[26] a well-known quantity over the temperature range of interest, i.e., from about 1000°K–2500°K, is dominant. Thus the emphasis in this study is on those parameters which influence electron densities.

In Fig. 5 it is seen that reducing k_{17} by a factor of 10 has a drastic effect on the calculations; a small additional reduction in attenuation is achieved via use of $k_{17}/100$.* Note that all three values of k_{17} are within the error bounds cited in Table 1. A reduction in the D/G eddy viscosity coefficient $\bar{K}$ by a factor of 1.5 is seen to displace the peak attenuation downstream and increase the electrical plume length. In Fig. 6 we observe that changing k_{17} by a factor of 10 for system B* has a smaller effect on attenuation than it had for system *A*. If measured attenuation for this system were to

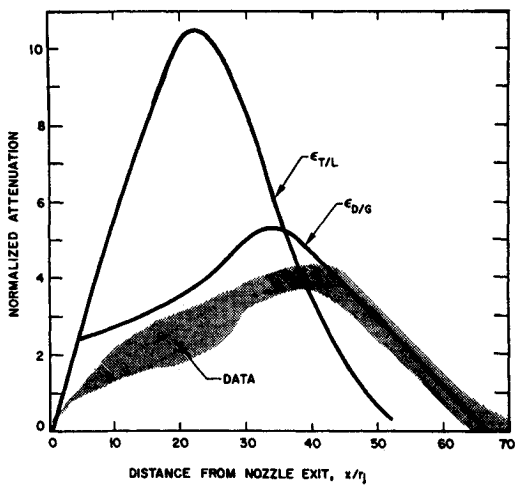

Fig. 7 Comparison between transverse attenuation calculations and test data, sea level, System C. All rate coefficients as listed in Table 1; Pr_T = 1.0.

* Computer time limitations prevented completion of this calculation.

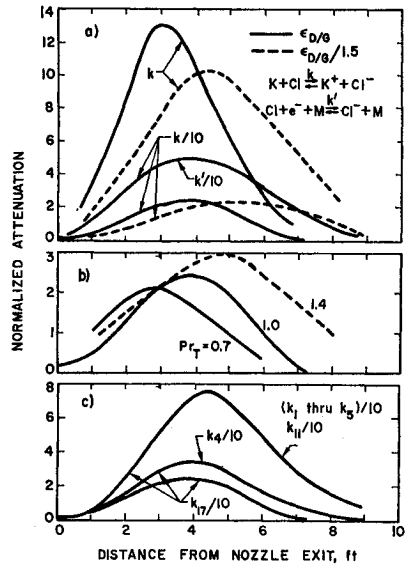

Fig. 8 Influence of various parameters on attenuation calculations, 28 kft, System A. a) charged species rate coefficients. All coefficients as listed in Table 1, except as noted; Pr_T = 1.0. b) turbulent Prandtl number. All rate coefficients as listed in Table 1, except $k_{17}/10$; $\epsilon_{D/G}$. c) 3-body radical recombination rate coefficients. All coefficients as listed in Table 1, except as noted; Pr_T = 1.0; $\epsilon_{D/G}$.

fall within the two predictions, both could be said to "agree" with the data, since the prediction of radar attenuation to within a factor of 2 is considered good. From the results shown in Fig. 7, for system *C* (the only system for which theory and experiment could be compared), some confidence is gained in the parallel mixing and afterburning model and in the D/G eddy viscosity formulation.† Use of the T/L model is seen to give results which overestimate the peak attenuation and underestimate the electrical plume length.

Results of parametric calculations for System *A* at 28 kft are shown in Fig. 8. In Fig. 8a we note that: 1) when the recommended value of k_{17} is used, a reduction in $\epsilon_{D/G}$ by a factor of 1.5 only reduces the peak attenuation by about 25%; however, the peak value is shifted further downstream, 2) an order of magnitude reduction in k_{17} has no effect on the predicted electrical plume length; however, the peak attenuation is reduced much more than at sea level (Fig. 5), 3) a decrease in $\epsilon_{D/G}$ by a factor of 1.5 has no effect on the peak at-

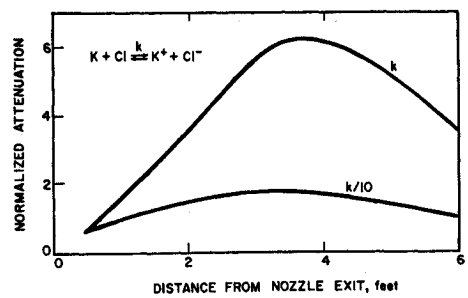

Fig. 9 Influence of ionization rate coefficient on attenuation calculations, 28 kft, System B. All rate coefficients as listed in Table 1, except as noted; $\epsilon_{D/G}$; Pr_T = 1.0.

† The use of still other eddy viscosity formulations[5] in these calculations could also result in reasonable agreement between attenuation predictions and measurements. Therefore, the results shown in Fig. 7 *do not* imply that the D/G model is "correct" for all afterburning rocket plumes.

tenuation when $k_{17}/10$ replaces k_{17}, and 4) an order of magnitude reduction in k_{18} does influence the predicted attenuation —increasing the peak values by a factor of about 2. However, there are indications[27] that the recommended value of k_{19} is too low and that reaction (19) is closer to equilibrium than these calculations suggest. Thus, if k_{19} were increased the apparent influence of reaction (18) on predicted attenuation would be reduced.

Figure 8b shows that varying Pr_T from 0.7–1.4 does significantly influence the trend of attenuation as a function of axial distance. Experimental studies are needed to determine the best choice of Pr_T to use in calculations on flows with combustion.

Figure 8c illustrates the effects of uncertainties in the rate coefficients for the three-body radical recombination reactions (i.e., k_1–k_5 and k_{11}) on predicted attenuation. Simultaneous reduction in the k's by a factor of 10 has a relatively large effect on attenuation, but very little change occurs when only k_4 is reduced. Hence reaction (4) is not a dominant reaction, although it does play a significant part in determining transverse attenuation. Further computer and laboratory studies are needed to determine the relative contributions made by the several radical recombination reactions.

Figure 9 shows a larger reduction in attenuation for system B at 28 kft than at sea level (Fig. 6), when k_{17} is reduced by a factor of 10. This is consistent with the results for system A.

Concluding Remarks

These analytical results provide a framework by which attenuation data, taken transverse to afterburning solid-propellant plumes with focused microwaves, can be interpreted. A single comparison between theory and experimental data (at sea level) has shown that the parallel mixing model is adequate. For this system, the use of the Donaldson/Gray eddy viscosity formulation gave better agreement with the data than the use of the Ting/Libby formulation.

Realistic calculations of solid-propellant exhaust plume electrical properties, and the interpretation of simulated altitude attenuation data, must take chemical kinetic effects into account. Calculations based on local thermochemical equilibrium can be considerably in error.

For more accurate predictions of the plume electrical properties, at sea level and altitude, it is particularly important to know: 1) the rate coefficient for the dominant charge-producing reaction, $K + Cl \rightarrow K^+ + Cl^-$, to better accuracy, 2) whether the Donaldson/Gray eddy viscosity model gives a good representation of momentum transport in flows with combustion, 3) the relation between momentum, mass and heat transport rates in flows with combustion; i.e., is Pr_T less than unity (as measured in nonreactive flows) in flows with combustion, and 4) the rate coefficients for the reactions governing decay of radial concentrations to better accuracy.

Appendix: Sources of Rate Coefficient Data for Charged Species Reactions‡

$$K^+ + e^- + M \rightleftarrows K + M, \qquad k_{16} = 2 \times 10^{-22} T^{-1.5}$$

This rate coefficient was calculated from measurements on the reverse reaction by Jensen and Padley.[28,29] The corresponding value for atmospheric-pressure $CO/N_2/O_2$ flames is $1.5 \times 10^{-22} T^{-1.5}$

$$K^+ + Cl^- \rightleftarrows K + Cl, \qquad k_{17} = 1 \times 10^{-8} T^{-0.5}$$

This rate coefficient was measured by Hayhurst[30] and reported by Sugden and Hayhurst.[31] Note that Baulknight and Bortner[32] estimate the following theoretical rate coefficients:

$$O^+ + O^- \rightarrow O + O, \qquad k = 5.5 \times 10^{-7} T^{-0.5}$$

$$N^+ + O^- \rightarrow N + O, \qquad k = 8.4 \times 10^{-7} T^{-0.5}$$

Comparison with these estimates led us to insert a $T^{-0.5}$ temperature dependence in k_{17}; Sugden[33] reports that the measured temperature dependence is slight

$$Cl + e^- + M \rightleftarrows Cl^- + M, \qquad k_{18} = 3 \times 10^{-30}$$

This rate coefficient is taken to be the same as that for $O_2 + e^- + M \rightarrow O_2^- + M$, which is based on measurements made at about 500°K by Pack and Phelps.[34,35] These authors found that k increased slightly with increasing temperature, which is perhaps contrary to what one might expect. We have used a temperature-independent value for $1000 < T < 3500$°K. The data on third-body relative efficiencies are meager; Pack and Phelps find that H_2O is about four times as efficient as O_2. For a review of attachment of electrons to O_2, see Ref. 36

$$HCl + e^- \rightleftarrows H + Cl^-, \qquad k_{19} = 1 \times 10^{-10} \exp -20{,}000/RT$$

This rate coefficient was estimated by Calcote and Jensen[37] on the basis of collision cross section data obtained by Buchel'nikova.[38] Note, however, that results of Fehsenfeld, Ferguson, and Schmeltekopf[27] suggest that the reaction may be considerably faster.

‡ Taken from Ref. 12.

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